From Galilean Covariance to Gauge Conditions: A Thermodynamic Insight to Signal Integrity

Loïc Rondot[1], Thiago H. Borges[2] and Vincent G. Mazauric[2]
[1]CEDRAT, 15 chemin de Malacher, 38246 – Meylan, France
[2]Schneider Electric, Strategy & Innovation, 38TEC, 30050 – Grenoble Cedex 9, France
E-mail: vincent.mazauric@schneider-electric.com

***Abstract* — In order to address the signal integrity issue, Galilean electromagnetism is derived from a thermodynamic approach. Attention is paid on the various regimes allowed by the quasi-static limit. It is emphasized that an abrupt transition exists between the QS-magnetic and the QS-electric regimes for which different gauge conditions on the potentials should be considered.**

I. INTRODUCTION

Loss of signal integrity becomes a critical issue in power electrical engineering, due to the higher level of integration in power electronics, the generalization of Pulse Width Modulation power supply, or the decreasing duration of commutations. In order to address their considerable impacts on both energy efficiency and the performance requirements former analyses were mostly focused on the circuit description. Loss in signal integrity is then explained as a more or less continuous change in the impedance of the system with the frequency. At the opposite, the problem is here addressed within the framework of the Galilean electromagnetism introduced in [1]. It is shown that the loss of signal integrity must be considered as intrinsically discontinuous.

In the following, a variational approach of electromagnetism is proposed. Then various regimes occurring within the quasi-static limit are discussed from the covariance property. Finally, the transition between regimes is explained as a change in the gauge condition.

II. VARIATIONAL FORMULATION OF ELECTROMAGNETISM

Classically, thermodynamic approaches of electromagnetism do not consider any extension towards time-varying regimes [2]. Whereas some improvements are summarized in [3] for steady state regimes, no general contribution is available for transient. Denoting, as a general rule in this work, variational parameters or functionals thanks to *italic* fonts whereas roman ones specify their values at the minimum, the magnetodynamic behavior of any electrical system is derived from the functional [4]:

$$-\mathrm{P}_{\mathrm{mech}}-\frac{\mathrm{dG}}{\mathrm{d}t}=\min_{\boldsymbol{H},\boldsymbol{E}}\left(\int_C \sigma^{-1}(\mathbf{curl}\boldsymbol{H})^2\,\mathrm{d}^3r+\frac{\mathrm{d}}{\mathrm{d}t}\int(\mathbf{B}\cdot\boldsymbol{H}+\mathbf{D}\cdot\boldsymbol{E})\mathrm{d}^3r\right) \quad (1)$$

where the functional in the RHS exhibits:

- the magnetic field $\boldsymbol{H}$ related to free and displacement currents according to the Maxwell-Ampere equation:
$$\mathbf{curl}\boldsymbol{H}=\boldsymbol{J}+\partial_t\boldsymbol{D} \quad (2)$$
While the quasi-static approximation enforces a vanishing electric displacement ($\mathbf{D}\equiv 0$) in conductors (i.e. no free charges), the charge conservation at the various interfaces provides the continuity equation:
$$\mathbf{n}\times([\boldsymbol{H}]-\mathbf{V}\times[\boldsymbol{D}])=0 \quad (3)$$
where $\mathbf{V}$ is the velocity of the interfaces in the rest frame, $\mathbf{n}$ the unit vector oriented by the interface and $[\cdot]$ denotes the field discontinuity occurring thereon;
- the Joule losses P_J monitored in conductors. This term is even to respect invariance of losses with the inversion of time (σ^{-1} is the resistivity);
- the variation with time of the electromagnetic energy coupling the field with the generator I and the mass V_0;
- the magnetic $\mathbf{B}(\boldsymbol{h})$ and electrostatic $\mathbf{D}(\boldsymbol{e})$ behavior laws derived from thermostatic equilibrium of the Gibbs potential:

$$G(\mathrm{I},\mathrm{V}_0)=G_{\mathrm{m}}+G_{\mathrm{e}}=\int\left(\int_0^{\boldsymbol{H}}(-\boldsymbol{B})\cdot\delta\boldsymbol{h}+\int_0^{\boldsymbol{E}}(-\boldsymbol{D})\cdot\delta\boldsymbol{e}\right)\mathrm{d}^3r \quad (4)$$

where the flux density $\mathbf{B}$ and the electric displacement $\mathbf{D}$ are divergence-free to ensure that G is a state-function.

Galilean covariance states that stationary conditions expressed from (1) do adopt a form independent of the Galilean frame where is performed the time-derivation. Introducing $\mathbf{V'}$ as the relative velocity of the frame (’) in (1), some calculations on the convective derivative of the electromagnetic energy coupling yield the transformation law for the electric field:

$$\mathbf{E'}=\mathbf{E}+\mathbf{V'}\times\mathbf{B} \quad (5)$$

whereas the flux density and the electric displacement are kept:

$$\mathbf{B'}=\mathbf{B} \quad (6)$$

$$\mathbf{D'}=\mathbf{D} \quad (7)$$

Hence, extending the electric field in a moving conductor (with the velocity $\mathbf{V}$) according to Ohm’s law with motion:

$$\mathbf{E}=\sigma^{-1}\mathbf{J}-\mathbf{V}\times\mathbf{B} \quad (8)$$

Maxwell-Faraday’s equation:

$$\mathbf{curl}\,\mathbf{E}=-\partial_t\mathbf{B} \quad (9)$$

and its subsequent continuity equation:

$$\mathbf{n}\times([\mathbf{E}]+\mathbf{V}\times[\mathbf{B}])=0 \quad (10)$$

may be viewed as acting locally to check globally a tendency towards reversibility [5]. This striking property provides a thermodynamic oriented insight of the variational theory of electromagnetism [6]. Hence, the functional (1) balances the variations with time of the co-energy (–G) and the mechanical power received from the field by the actuators ($-\mathrm{P}_{\mathrm{mech}}$).

It should be noticed that Maxwell-Faraday equation (9) provides the flux density divergence-free at any time which is a mandatory condition to derive (5) in its usual form. Similarly, the transformation law of the magnetic field reads:

$$\mathbf{H'}=\mathbf{H}-\mathbf{V'}\times\mathbf{D} \quad (11)$$

according to (3).

In order to consider sub-systems for design purpose, it is convenient to introduce the electrical power on the domain Ω:

$$P_{\text{elec}}(\Omega) = -\oint_{\partial\Omega} (\boldsymbol{E} \times \boldsymbol{H}) \cdot \mathbf{n}\, \mathrm{d}^2 r \quad (12)$$

so that the contribution of Ω to (1) reads:

$$-\mathrm{P}_{\text{mech}}(\Omega) + \mathrm{P}_{\text{elec}}(\Omega) - \frac{\mathrm{dG}}{\mathrm{d}t}(\Omega) \quad (13)$$

III. QUASI-STATIC REGIMES

The set of transformation laws (6), (7), (5) and (11) is not compatible with the invariance of the behavior laws, at least in vacuum. As quasi-static phenomena are observed, it is mandatory to admit that one coupling term in (1) prevails much more than the other. As a result, the quasi-static limit must be split in magnetic and electric regimes (Tab. 1) [1]. It should be noticed that the conductors are always depicted within the magnetic regime of the quasi-static limit.

TABLE 1. REGIMES ALLOWED WITHIN THE QUASI-STATIC LIMIT (FROM [1]). THE RESOLUTION IS PERFORMED IN TWO STEPS: THE FIRST (SECOND) ONE INVOLVES THE DOMINANT (MARGINAL) COUPLING. THE ARROW (⇐) DENOTES A FIELD OBTAINED FROM THE MAIN RESOLUTION WHICH ACTS AS A SOURCE FOR THE SECOND ONE.

	magnetic regime $\lvert \mathbf{B}\cdot\mathbf{H} \rvert >> \lvert \mathbf{D}\cdot\mathbf{E} \rvert$	electric regime $\lvert \mathbf{B}\cdot\mathbf{H} \rvert << \lvert \mathbf{D}\cdot\mathbf{E} \rvert$
conductors	$\mathbf{curl}(\sigma^{-1}\mathbf{curl}\mathbf{H}) = -\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$, $\operatorname{div}\mathbf{B} = 0$, $\mathbf{curl}\,\mathbf{H} = \mathbf{J}$, $\mathbf{B} = \mu\mathbf{H}$	N/A
dielectrics	$\operatorname{div}\mathbf{B} = 0$, $\mathbf{curl}\mathbf{H} = 0$, $\mathbf{B} = \mu\mathbf{H}$	$\operatorname{div}\mathbf{D} = 0$, $\mathbf{curl}\mathbf{E} = 0$, $\mathbf{D} = \varepsilon\mathbf{E}$
dielectrics	$\mathbf{curl}\,\mathbf{E} \Leftarrow -\partial_t\mathbf{B}$, $\operatorname{div}\mathbf{D} = 0$, $\mathbf{D} = \varepsilon\mathbf{E}$, $(\partial C)\ \mathbf{n}\times\mathbf{E} \Leftarrow \mathbf{n}\times(\sigma^{-1}\mathbf{J} - \mathbf{V}\times\mathbf{B})$	$\mathbf{curl}\,\mathbf{H} \Leftarrow \partial_t\mathbf{D}$, $\operatorname{div}\mathbf{B} = 0$, $\mathbf{B} = \mu\mathbf{H}$, $(\partial C)\ \mathbf{n}\times[\mathbf{H}] \Leftarrow \mathbf{n}\times(\mathbf{V}\times\mathbf{D})$
Contribution to the functional (13)	$-\mathrm{P}_{\text{mech}} + \mathrm{P}_{\text{elec}} - \frac{\mathrm{dG}_{\text{m}}}{\mathrm{d}t}$ (14)	$-\mathrm{P}_{\text{mech}} + \mathrm{P}_{\text{elec}} - \frac{\mathrm{dG}_{\text{e}}}{\mathrm{d}t}$ (15)

Hence, the signal integrity issue in Electro-Magnetic Compatibility may be viewed as a transition in (1) between the magnetic and the electrostatic couplings in a given area of the dielectric region, the actual regime corresponding to the most reversible evolution of the *global* system.

IV. GAUGE CONDITIONS

As a general rule, phase transition is associated with a break in the symmetry of the problem [7]. This point is discussed from the magnitude of the various times occurring in any electrodynamic problem of size ℓ [8], namely the magnetic diffusion time $\tau_{\text{m}} = \sigma\mu\ell^2$ and the charge life duration $\tau_{\text{e}} = \frac{\varepsilon}{\sigma}$ occurring in conductor; and the transit time of an electromagnetic wave crossing the system $\tau_{\text{em}} = \sqrt{\varepsilon\mu}\,\ell$ checking the relation $\tau_{\text{em}}^2 = \tau_{\text{m}}\tau_{\text{e}}$. These values could be faced to the typical duration τ of the excitation so that the various regimes may be schematically stressed (Fig.1) [9].

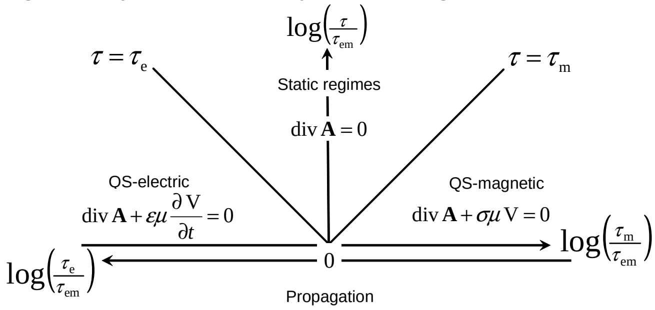

Fig. 1. Domains of validity of the Quasi-static regimes with their gauges conditions. Notice the absence of border between the electric and magnetic regimes which reinforces the existence of a transition driven by the gauges.

Defining the flux density and the electric field from the potentials (**A**,V) according to:

$$\begin{aligned} \mathbf{B} &= \mathbf{curl}\,\mathbf{A} \\ \mathbf{E} &= -\mathbf{grad}\,\mathrm{V} - \partial_t\mathbf{A} \end{aligned} \quad (16)$$

various gauges may be considered according to the quasi-static regime to decouple the evolution equations expressed on the potentials. As a result, the transition between the QS-magnetic and the QS-electric regimes corresponds respectively to a change from the Stratton's to the Lorentz' gauges driven by the frequency $\omega = \frac{2\pi}{\tau}$ [10].

V. CONCLUSION AND FORTHCOMING

The loss of signal integrity is explained as a transition between the QS-magnetic and QS-electric regimes. Moreover, thermodynamics provides the functionals (14) and (15) from which the discussion should be achieved. Such an implementation should be highly valuable to discuss signal integrity issue.

VI. REFERENCES

[1] M. Le-Bellac and J.-M. Lévy-Leblond, "Galilean electromagnetism," *Il Nuovo Cimento*, vol. 14B, pp. 217-233, 1973.

[2] L. D. Landau and E. M. Lifschitz, *Electrodynamics of continuous media*. New York, NY, USA: Pergamon, 1960.

[3] T. Christen, "Application of the maximum entropy production principle to electrical system," *Journal of Physics D: Applied Physics*, vol. 39, pp. 4497-4503, 2006.

[4] V. Mazauric, "From thermostatistics to Maxwell's equations: A variational approach of electromagnetism," *IEEE Transactions on Magnetics*, vol. 40, pp. 945-948, 2004.

[5] V. Mazauric and N. Maïzi, "A global approach of electromagnetism dedicated to further long-term planning," *Proceedings in Applied Mathematics and Mechanics*, vol. 7, pp. 2130003-2130004, 2007.

[6] P. Hammond, *Energy methods in electromagnetism*. New York, USA: Oxford University Press, 1981.

[7] J. M. Yeomans, *Statistical mechanics of phase transitions*. Oxford, UK: Oxford University Press, 1992.

[8] G. Rousseaux, "Lorenz or Coulomb in Galilean electromagnetism," *Europhysics Letters*, vol. 71, pp.15-20, 2005.

[9] J. R. Melcher, *Continuum Electromechanics*, Cambridge, MA, USA: The MIT press, 1981.

[10] J. A. Stratton, *Electromagnetic Theory*, New York, NY, USA: Mac Graw Hill, 1941.